Published by Institute of Physics Publishing for SISSA
Received: *February 26, 2008*
Accepted: *April 1, 2008*
Published: *April 11, 2008*

OPE-RχT matching at order α_s: hard gluonic corrections to three-point Green functions

Matthias Jamin
Institució Catalana de Recerca i Estudis Avançats (ICREA), IFAE, Theoretical Physics Group, UA Barcelona, E-08193 Bellaterra, Barcelona, Spain
E-mail: jamin@ifae.es

Vicent Mateu*
Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, D-80805 München, Germany
E-mail: mateu@mppmu.mpg.de

Abstract: In this work we push the matching between the QCD operator product expansion (OPE) and resonance chiral theory (RχT) to order α_s. To this end we compute two- and three-point QCD Green functions (GFs) in both theories and compare the results. GFs which are order parameters of chiral symmetry breaking make this matching more transparent and thus we concentrate on those. On the OPE side one needs to calculate the hard-gluon virtual corrections to the quark condensate, and in particular for three-point GFs this computation was hitherto missing. We also discuss the need for including the infinite tower of hadronic states in the hadronic representation of the GF when non-analytic terms such as logarithms are present in the OPE Wilson coefficients.

Keywords: 1/N Expansion, NLO Computations, QCD, Chiral Lagrangians.

*On leave from: Departament de Física Teòrica, IFIC, Universitat de València-CSIC, Apt. Correus 22085, E-46071 València, Spain.

Contents

1. Introduction

Green functions (GFs for short) of colour singlet local QCD currents have proved to be an essential tool for a successful description of hadronic physics. Confinement binds quarks and gluons into colour singlet hadrons and hence it makes little sense to compute matrix elements with those fundamental particles as asymptotic states. The process of hadronisation takes place at an energy scale of the order of Λ_{QCD} and thus it is essentially non-perturbative. QCD currents can be written as

$$J^a_\Gamma(x) \equiv\, :\bar{q}_{\alpha i A}(x)\Gamma_{\alpha\beta}\left(\frac{\lambda^a}{2}\right)_{ij} q_{\beta j A}(x): \,=\, :\bar{q}(x)\,\Gamma\,\frac{\lambda^a}{2}\,q(x):\,, \tag{1.1}$$

where Γ and λ^a are spin and flavour matrices, respectively, and A denotes a (summed) colour index. The Γ matrix stands for $2\,I$, $2\,i\,\gamma_5$, γ^μ, $\gamma^\mu\,\gamma_5$ and $\sigma^{\mu\nu}$ in the case of scalar, pseudoscalar, vector, axial-vector and tensor sources, respectively.[1] The currents $J^a_\Gamma(x)$ qualify as interpolating fields for mesonic states H^a_Γ in the sense of having non-vanishing matrix elements between the vacuum and the hadronic state: $\langle\, H^a_J \,|\, J^b_\Gamma \,|\, 0\,\rangle \neq 0$. This

[1] In our conventions $\gamma_5 = i\,\gamma_0\gamma_1\gamma_2\gamma_3$ and $\sigma^{\mu\nu} = \frac{i}{2}\,[\gamma^\mu, \gamma^\nu]$.

permits to obtain matrix elements of hadronic states from GFs through the LSZ reduction formula. Let us define the two- and three-point GFs in momentum space:

$$
\begin{aligned}
\Pi^{ab}_{12}(p) \;&\equiv\; i\int \mathrm{d}^4x\, e^{ip\cdot x}\langle\, 0\,|\, T\{J^a_1(x)J^b_2(0)\}|\, 0\,\rangle\,, \\
\Pi^{abc}_{123}(p,q) \;&\equiv\; i^2\int \mathrm{d}^4x\,\mathrm{d}^4y\, e^{i(p\cdot x+q\cdot y)}\langle\, 0\,|\, T\{J^a_1(x)J^b_2(y)\,J^c_3(0)\}|\, 0\,\rangle\,.
\end{aligned} \tag{1.2}
$$

These definitions can be trivially generalised for more currents.

QCD exhibits a spontaneous breakdown of the chiral symmetry due to quantum effects. It is believed that the operator responsible for this phenomenon is the quark condensate $\langle\bar{q}q\rangle$, which acquires a non-vanishing expectation value. Phenomenologically, this feature is reflected through the appearance in the spectrum of the so called pseudo-Goldstone bosons. At very low energies (long distances) $E \ll m_\rho$, GFs can be computed as a perturbative series in the (small) momentum carried by the current and the quark masses [1],[2] in the so called Chiral Perturbation Theory (χPT) [2, 3], which constitutes a low-energy effective field theory (EFT). In this formalism the non-perturbative effects are encoded in the a priori unspecified low-energy constants (LECs) that grow in number as we go further in the chiral expansion.

At high energies (short distances) the QCD currents in the GF approach each other and one can expand their product as a tower of local operators in the Wilson operator product expansion (OPE) [4]. In momentum space this is tantamount to an expansion in inverse powers of momenta, only being valid in the deep Euclidean region $-p^2 \gg m_\rho^2$. The vacuum expectation values of these local operators are known as vacuum condensates and encode the non-perturbative effects [5, 6], which start to become relevant at intermediate scales. Then we have an explicit separation of the short distance effects in the Wilson coefficients and the long distance effects in the condensates. The arbitrariness of this splitting is reflected in the dependence of both the Wilson coefficients and the condensates on the renormalisation scale μ and the employed renormalisation scheme.

At intermediate energies the most reliable tool is an expansion of QCD in powers of $1/N_C$ [7−9], N_C being the number of colours of the QCD gauge group SU(N_C). This region is populated by hadronic resonances driving the strong dynamics. At leading order in $1/N_C$ there must be an infinite number of such states for any given set of quantum numbers. GFs of a finite subset of these resonances can be obtained as tree-level diagrams of an effective Lagrangian with resonances as explicit degrees of freedom, known as RχT [10, 11]. Nevertheless, we lack a method to handle an infinite tower of such states, and then the so called minimal hadronic ansatz (MHA) is usually used, meaning that we only consider the lowest lying resonances. One is then able to perform a matching of the three regimes and thus to estimate the values of LECs present in the χPT Lagrangian [12, 13].[3]

The power of EFTs is enhanced by matching onto the more fundamental theory in a region where both descriptions are sensible, and running down with the renormalisation

[2]Throughout this paper we will assume the chiral limit of QCD, that is we will set the masses of the light quarks u, d and s to zero.

[3]An approach in the framework of Padé approximants to approximate GFs in the large-N_C limit which encompasses saturation with a finite number of resonances, can be found in ref. [14].

group equation (RGE) to a scale where the EFT is useful. This results in a resummation of large logarithms that would otherwise spoil the perturbative expansion. As hadronic theories, the matching between χPT and RχT is straightforward since up to a certain mass scale Λ_χ the spectra of both EFTs coincide, and the matching can be accomplished be formally integrating out the heavy degrees of freedom. However, the situation changes drastically when matching RχT onto QCD, because the spectrum no longer has any particle in common. The reason for it is that we are going through the chiral phase transition. Then one is forced to compare GFs computed in the different EFTs in momentum regions where their domains of validity overlap.

The matching relation is better understood for GFs which are order parameters of chiral symmetry breaking (order parameters for short). These GFs have a vanishing Wilson coefficient for the identity operator (that is, the purely perturbative contribution) to all orders in α_s in the chiral limit, and hence its leading contribution stems from the $\langle \bar{q}q \rangle$ condensate. Since this operator is also responsible for the chiral symmetry breaking those GFs encode essential information on its mechanism.

This method has been used for two-point GFs including several orders of α_s corrections [15], but only at leading order for three-point GFs [16 – 20] (see also [21] for a matching with two multiplets of vector-meson resonances). It is the purpose of this paper to push the matching up to $\mathcal{O}(\alpha_s)$. Unfortunately, the $\mathcal{O}(\alpha_s)$ corrections to the $\langle \bar{q}q \rangle$ Wilson coefficient $C_{\langle \bar{q}q \rangle}$ for three-point GFs are not known, and we will concentrate on their computation, relegating the details of the matching to a forthcoming paper. Finally, for the case of the $\Pi_{\rm VT}$ GF, we will show that we can perform the matching to the one-loop OPE result with a single multiplet of vector-meson resonances.

The paper is organised as follows: In section 2 we discuss the renormalisation scale dependence of the GFs and how this affects the matching; in section 3 we compute the $\mathcal{O}(\alpha_s)$ corrections to the OPE two-point GFs and match the $\langle VT \rangle$ result onto RχT with only one multiplet of vector-meson resonances; in section 4 we outline the calculation of the $\mathcal{O}(\alpha_s)$ corrections to the OPE three-point GFs and argue what the difficulties are when matching onto the hadronic representation; in section 5 we present our conclusions.

2. Scale dependent Green functions

A strong motivation for computing the $\mathcal{O}(\alpha_s)$ corrections to $C_{\langle \bar{q}q \rangle}$ is to become sensitive to the renormalisation dependence of the quark condensate in full QCD. As is well known, since a QCD current involves two quark fields in the same space-time point, it is not enough to renormalise the quark fields to get a finite result. The current itself must be renormalised in addition:

$$J_\Gamma^B \equiv Z_\Gamma \, J_\Gamma \, , \tag{2.1}$$

which defines the renormalisation constant Z_Γ. The superscript B denotes μ-independent bare currents while currents without superscripts correspond to renormalised (hence generally μ-dependent) ones, where μ signifies the arbitrary renormalisation scale. It is customary to express the μ-dependence through the anomalous dimension γ_Γ, which is defined as

$$\gamma_\Gamma \equiv \frac{\mu}{Z_\Gamma}\frac{\mathrm{d}Z_\Gamma}{\mathrm{d}\mu}, \qquad -\mu\,\frac{\mathrm{d}J_\Gamma}{\mathrm{d}\mu} = \gamma_\Gamma\, J_\Gamma\,. \tag{2.2}$$

The anomalous dimension depends on the coupling $a_s \equiv \alpha_s/\pi$, and in perturbation theory has an expansion

$$\gamma_\Gamma(a_s) = \gamma_\Gamma^{(1)}\, a_s + \gamma_\Gamma^{(2)}\, a_s^2 + \gamma_\Gamma^{(3)}\, a_s^3 + \cdots\,. \tag{2.3}$$

At leading order in α_s one can easily calculate the anomalous dimension of a current with the following master formula (of course, anomalous dimensions are gauge invariant):

$$\gamma_\Gamma^{(1)}\,\Gamma = \frac{C_F}{2}\,\Gamma - \frac{C_F}{8}\,\gamma^\mu\,\gamma^\nu\,\Gamma\,\gamma_\nu\,\gamma_\mu\,, \tag{2.4}$$

where the algebra should be performed in *four* space-time dimensions. Plugging in the structures for the different Γ's one finds that vector V_μ^a and axial-vector A_μ^a currents have zero anomalous dimension. This is a general result that stems from the fact that in the chiral limit both currents are conserved. In the $\overline{\mathrm{MS}}$ scheme which we follow in this paper the renormalisation procedure is mass independent and so the result for the anomalous dimensions holds for finite quark masses as well. Similarly, one can show that the quantities $m_q{\cdot}S^a$ and $m_q{\cdot}P^a$ are μ independent, if the currents are normal-ordered. Whence it follows that

$$\gamma_S = \gamma_P = \frac{\mu}{m_q}\frac{\mathrm{d}m_q}{\mathrm{d}\mu} \equiv -\gamma_m\,, \tag{2.5}$$

to all orders in α_s. The μ dependence of the renormalised current is reflected in the GFs themselves. We can write the dependence through a RGE[4]

$$\left[\mu\,\frac{\partial}{\partial\mu} + \sum_{i=1}^{n}\gamma_i\right]\Pi_{1\cdots n} = 0\,. \tag{2.6}$$

Here, γ_i are the anomalous dimensions corresponding to the currents appearing in $\Pi_{1\cdots n}$. QCD vacuum condensates in general also have a non-vanishing anomalous dimension, and in particular for the quark condensate $\langle\bar{q}q\rangle$, as should be clear from the above, $\gamma_{\langle\bar{q}q\rangle}$ coincides with γ_S of eq. (2.5).[5]

Let us now consider the case of the 3-point Green function Π_{SSS} with three scalar currents. In the OPE it gets its first contribution from $\langle\bar{q}q\rangle$, and at leading order its Wilson coefficient is μ-independent. On the RχT side the situation is a bit different. It is well known that each scalar and pseudoscalar current insertion in the chiral theory is accompanied by a $\langle\bar{q}q\rangle$ factor, so [19]

$$\Pi_{SSS}^{\mathrm{OPE}} = C_{\langle\bar{q}q\rangle}^{SSS}(p,q)\,\langle\bar{q}q\rangle(\mu)\,, \qquad \Pi_{SSS}^{\mathrm{R}\chi\mathrm{T}} = \widetilde{\Pi}(p,q)\,[\langle\bar{q}q\rangle(\mu)]^3\,, \tag{2.7}$$

[4]This argument is spoiled if the GF needs subtractions, that is it does not correspond to a physical quantity. Since we are dealing with order parameters, our GFs do not need any subtraction.

[5]There exists a sophistication in that for a proper separation of long and short distances in the OPE, non-normal-ordered condensates should be employed, but then $m\langle\bar{q}q\rangle$ is not RG-invariant [22 – 24]. However, here we are working in the chiral limit, and for vanishing quark masses this problem is absent.

and on first sight, we cannot match one onto the other because the μ dependences appear different. What is happening is that RχT includes all α_s orders in a non-perturbative fashion, while in QCD the coefficient function $C^{SSS}_{\langle\bar{q}q\rangle}$ must exhibit some μ dependence to account for the missing scale factors. This situation easily generalises to other Green functions, and we can write a RGE for the coefficient function $C_{\langle\bar{q}q\rangle}(\mu)$, such that eq. (2.6) is fulfilled:

$$\left[\mu\frac{\partial}{\partial\mu}+\gamma_m+\sum_{i=1}^{n}\gamma_i\right]C_{\langle\bar{q}q\rangle}(\mu)=0\,. \tag{2.8}$$

Now the scale dependence of the Green functions in both theories of eq. (2.7) agrees and in principle the matching could be performed.

3. Two-point GFs and matching

To prepare the discussion of the 3-point functions below, we begin with reviewing the calculation of the α_s corrections for $C_{\langle\bar{q}q\rangle}$ in the case of two-point order parameter GFs. In this simpler scenario, we will discuss the appearance of infrared (IR) divergences at intermediate stages of the calculation and the renormalisation of $\langle\bar{q}q\rangle$. For 2-point functions in which the quark condensate is the leading contribution in the chiral limit, there are only two GFs to be considered: $\Pi^{\mu}_{\rm AP}$ and $\Pi^{\mu,\nu\alpha}_{\rm VT}$.

The fact that vector and axial-vector currents are conserved in the chiral limit has implications on the GFs, known as Ward identities. For the case of the vector-tensor GF they imply that $p_\mu\,\Pi^{\mu,\nu\alpha}_{\rm VT}(p)=0$, and so taking into account the antisymmetry of the tensor current, together with parity, we can parametrise it as

$$(\Pi_{\rm VT})^{ab}_{\mu,\nu\rho}(p)=i\,\delta^{ab}\,(g_{\mu\nu}\,p_\rho-g_{\mu\rho}\,p_\nu)\,\Pi_{\rm VT}(p^2)\,. \tag{3.1}$$

For the $\Pi^{\mu}_{\rm AP}$ GF, Ward identities have somewhat deeper consequences and in fact completely fix it in the chiral limit:

$$(\Pi_{\rm AP})^{ab}_{\mu}(p)=2\,i\,\delta^{ab}\,\frac{p_\mu}{p^2}\,\langle\bar{q}q\rangle\ , \tag{3.2}$$

where a detailed derivation can be found in appendix A. Again, we see that the scale dependencies on both sides of eq. (3.2) are identical since the anomalous dimensions of the scalar current γ_S and of the quark condensate agree. Furthermore, one finds that in the chiral limit $\Pi_{\rm AP}$ is saturated by one pion exchange.

Let us now concentrate on the $\Pi^{\mu,\nu\alpha}_{\rm VT}$ GF. The relevant diagrams are shown in figure 1. Diagrams (c), (e) and (f) are infrared divergent since they involve gluons attached to quark lines with zero momentum. The same diagrams contribute to $\Pi_{\rm AP}$ and eq. (3.2) shows that it is free from IR divergences. So they cancel when adding the six diagrams, and the same occurs for the $\Pi_{\rm VT}$ GF. Since these divergences cancel at the end we might choose any method to regularise them. In what follows, we will adopt dimensional regularisation, which simplifies computations notoriously. In this scheme diagrams (e) and (f) are zero:

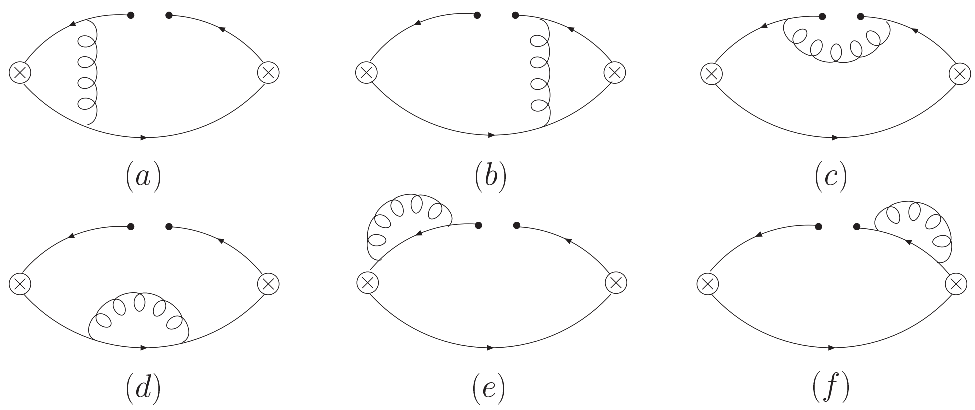

Figure 1: Gluonic corrections to the $C_{\langle\bar{q}q\rangle}$ for two-point GFs. Diagrams (c), (e) and (f) are infrared divergent.

they are scaleless and convert the IR divergence of diagram (c) into an ultraviolet (UV) one. In particular, for this diagram we obtain

$$\Pi_{\mathrm{VT}}^{(c)} = a_s \, \frac{C_F}{4} \, \frac{\langle\bar{q}q\rangle}{p^2} \left\{ -(3+a) \left[\frac{1}{\hat{\epsilon}} - \log\left(-\frac{p^2}{\mu^2} \right) \right] - 1 - a \right\}, \tag{3.3}$$

where a denotes the gauge parameter.[6] As we see, by itself this diagram is still gauge dependent, which means that the other diagrams are required in order to obtain a gauge-invariant result.

Summing up all diagrams, we find a remaining divergence of the form $-\,C_F\,a_s/\hat{\epsilon}$. Part of this divergence (namely $-\,3\,C_F\,a_s/4\,\hat{\epsilon}$) is absorbed in the renormalisation of the quark condensate, that is $\langle\bar{q}q\rangle^B = Z_{\langle\bar{q}q\rangle}\langle\bar{q}q\rangle(\mu)$, while the remaining divergence is removed with the renormalisation of the tensor current, $J_T^B = Z_T J_T(\mu)$, leading to

$$Z_T = 1 - \frac{C_F}{4} \, a_s \, \frac{1}{\hat{\epsilon}} + \mathcal{O}(a_s^2) \,. \tag{3.4}$$

From the renormalisation constant Z_T, we can also derive the first coefficient of the tensor-current anomalous dimension, $\gamma_T^{(1)} = C_F/2$, in agreement with the simple formula of Eq (2.4). For the full vector-tensor Green function at the next-to-leading order, we then find

$$\Pi_{\mathrm{VT}}^{\mathrm{OPE}}(p^2,\mu) = \frac{\langle\bar{q}q\rangle\,(\mu)}{p^2} \left\{ 1 + a_s\, C_F \left[\log\left(-\frac{p^2}{\mu^2} \right) - 1 \right] \right\} + \mathcal{O}(a_s^2, p^{-4}) \,, \tag{3.5}$$

which is of course gauge invariant. For N_C flavours $C_F = \frac{N_C^2-1}{2N_C} \approx \frac{N_C}{2}$ where the approximation corresponds to the large-N_C limit. With our result of eq. (3.5), we can check that the RGE for the coefficient function is trivially satisfied:

$$\left[\mu \, \frac{\partial}{\partial\mu} + \gamma_m + \gamma_T \right] C_{\langle\bar{q}q\rangle}^{\mathrm{VT}} = \left[\mu \, \frac{\partial}{\partial\mu} + 2\, a_s C_F \right] C_{\langle\bar{q}q\rangle}^{\mathrm{VT}} = 0 \,. \tag{3.6}$$

[6] All calculations in this paper have been performed in an arbitrary covariant gauge. The dependence on the gauge parameter a cancels in all our final, physical results, constituting a good check for them.

Let us now match this result onto the large-N_C prediction of RχT. Since the $\langle VT\rangle$ GF in the chiral limit does not need to be subtracted, it is completely determined by its spectral function. Then it can be regarded as an observable and thus we can directly match its expression in different approximations, such as OPE and RχT. However, we still have the problem that the tensor current requires renormalisation, and thus, the $\langle VT\rangle$ GF is scale and scheme dependent. This renormalisation dependence would then be reflected in a scale dependent coupling of the tensor current to vector mesons $f_V^T(\mu)$ on the hadronic side. Since we prefer to work with hadronic quantities which are explicitly scale independent, another possibility is to multiply $f_V^T(\mu)$ by an appropriate scale factor $R_T(\mu)$, which results in a scale independent tensor decay constant $\hat{f}_V^T$. This is analogous to the definition of scale-invariant B-factors, which parametrise hadronic matrix elements of four-quark operators, in the case of weak hadronic decays [25]. Therefore, we define

$$\begin{aligned}
\hat{f}_V^T \quad &\equiv \quad f_V^T(\mu)\, R_T(\mu) \;\equiv\; f_V^T(\mu)\, \exp\left\{ -\int^{a_s(\mu)} \frac{\gamma_T(a_s)}{\beta(a_s)}\, \mathrm{d}a_s \right\} \\
&= \quad f_V^T(\mu)\, [a_s(\mu)]^{-\gamma_T^{(1)}/\beta_1} \left[1 - \left(\frac{\gamma_T^{(2)}}{\beta_1} - \frac{\beta_2\, \gamma_T^{(1)}}{\beta_1^2} \right) a_s(\mu) + \mathcal{O}(a_s^2) \right] \qquad (3.7) \\
&\overset{N_f=3}{=} \quad f_V^T(\mu)\, [a_s(\mu)]^{-\frac{4}{27}} \left[1 - \frac{337}{486}\, a_s(\mu) + \mathcal{O}(a_s^2) \right] .
\end{aligned}$$

The anomalous dimension of the tensor current is known up to order a_s^3 [26, 27], and thus one could even extend eq. (3.7). However, at the order considered here this does not make sense, since we only stay at the next-to-leading order level. Now multiplying our result (3.5) for the Green function with the scale factor $R_T(\mu)$, it is a trivial exercise to convince oneself that $R_T(\mu)\, \Pi_{\mathrm{VT}}^{\mathrm{OPE}}(p^2,\mu)$ is scale independent at the considered order. Nevertheless, it should be kept in mind that it still depends on the renormalisation scheme, for example on the scheme in which $\langle \bar{q}q\rangle$ is renormalised.

In principle, since the next-to-leading order result for the $\langle VT\rangle$ GF contains a logarithm in the dynamical variable p^2, let us strongly emphasise that an infinite tower of resonances would be required for a sound matching to the RχT. Still, as a simple-minded approach, we will next consider the aforementioned minimal hadronic ansatz (MHA). This amounts to the assumption that a single resonance is enough to correctly describe the physics in a certain energy regime.[7] Tensor sources in chiral Lagrangians were first introduced in refs. [29]. In ref. [21] the matching for the $\langle VT\rangle$ GF was performed at $\mathcal{O}(\alpha_s^0)$, and here we will include the $\mathcal{O}(\alpha_s)$ corrections. Again multiplying the GF with the scale factor in order to obtain a scale-invariant quantity, the hadronic ansatz reads:

$$R_T(\mu)\, \Pi_{\mathrm{VT}}^{\mathrm{R}\chi\mathrm{T}}(p^2) = 2\, \frac{f_V\, \hat{f}_V^T\, m_V}{m_V^2 - p^2} \, . \qquad (3.8)$$

The precise definitions for the decay constants f_V and f_V^T can be found in eq. (53) of ref. [21]. eq. (3.8) is in principle assumed to be valid at all energies at leading order in

[7] For a discussion of the subtleties arising when matching the full tower of vector-meson resonances to the OPE for the $\langle VT\rangle$ GF the reader is referred to ref. [28].

$1/N_C$, since it incorporates chiral symmetry and the correct high-energy behaviour. It can be expanded in inverse powers of p^2, permitting a direct comparison with the OPE in eq. (3.5). However while eq. (3.8) is explicitly scale independent, (3.5) contains a logarithm which compensate the running of the tensor source and the quark condensate.

To perform the matching in practice, we choose a particular matching point and scale. First of all, to sum up the logarithm, we will employ the scale $\mu^2 = -p^2 \equiv M^2$. Then, M^2 should be large enough so that only keeping the first term in the OPE is a good approximation, while it should not be too large so that only putting one resonance on the hadronic side is reasonable. From these considerations, we would conclude, that M should be in the range 1–2 GeV. For the matching relation, we then find

$$f_V\, \hat{f}_V^T\, m_V = -\, 2\, [a_s(M^2)]^{-\frac{4}{27}\left\{\frac{3}{22}\right\}} \langle \bar{q}q\rangle(M^2) \left[1 - \frac{985}{486} \left\{ \frac{8357}{11616}\, N_C \right\} a_s(M^2) \right] , \qquad (3.9)$$

where in the curly brackets, we have also included the numbers corresponding to the large-N_C limit. eq. (3.9) can be viewed as a refinement over the analogous estimate of ref. [21].

Let us finally come to a numerical analysis of eq. (3.9). Employing the central values $f_V = 221\,\mathrm{MeV}$ and $M_V = 775\,\mathrm{MeV}$ [21], as well as the value for the quark condensate $\langle \bar{q}q\rangle(2\,\mathrm{GeV}) = -\,(267\,\mathrm{MeV})^3$ [24], and $\alpha_s(M_Z) = 0.119$, we obtain

$$\hat{f}_V^T = 195 \pm 55\ \mathrm{MeV}\,, \qquad (3.10)$$

where the quoted uncertainty dominantly results from a variation of the matching scale M in the range 1–2 GeV, and to a lesser extent from either taking the renormalisation-group coefficients in full QCD, or the large-N_C limit. The large matching-scale dependence of our result reflects the imperfection of the matching. At a scale of 1 GeV, the scale dependent vector-meson tensor coupling reads: $f_V^T(1\,\mathrm{GeV}) = 167 \pm 47\,\mathrm{MeV}$. Given the large uncertainties from the matching scale, this finding is in surprising agreement to the leading order result $f_V^T(1\,\mathrm{GeV}) = 165\,\mathrm{MeV}$ [21] and to determinations of the tensor coupling f_V^T from QCD sum rules and lattice QCD of refs. [30, 31].

4. Three-point GFs

Let us now concentrate on the main topic of this work: the α_s corrections to $C_{\langle \bar{q}q\rangle}$ for the three-point order-parameter GFs. Three-point GFs are of great interest because of several reasons. First, unlike the two-point case, there is a quite large amount of them that are order parameters of the chiral symmetry breaking. Second, in the framework of RχT they involve vertices between resonances and so they are useful for studying how the resonances interact. Third, there are many $\mathcal{O}(p^6)$ χPT LECs that can be determined with these GFs [20]. And fourth, by means of the LSZ reduction formula we can relate the GFs with form factors entering the calculation of many interesting hadronic observables. Some phenomenological applications of these Green functions can be found in ref. [32, 33] and references therein.

There are eight such GFs and we will sort them according to their intrinsic-parity and Lorentz tensor rank. For rank-two GFs we can use the chiral Ward identities and

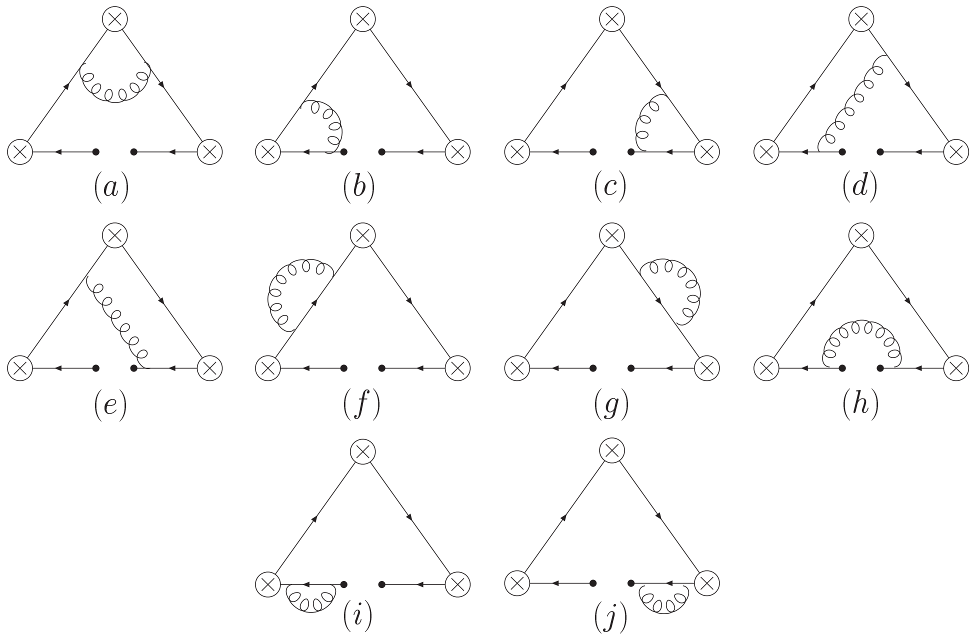

Figure 2: Gluonic corrections to $C_{\langle \bar{q}q\rangle}$ for three point GFs.

the transformation properties under parity for splitting them in terms of only one or two Lorentz tensors (made up of $g^{\mu\nu}$, $\varepsilon^{\mu\nu\alpha\beta}$ [8] and the external momenta). This amounts to a great simplification on their calculation. When two or more currents have the same Dirac structure one can also exploit Bose symmetry. We already saw that the flavour structure of two-point GFs is trivial: it is proportional to a Kronecker delta (there is no other SU(3) rank-two tensor). For the present case there are only two rank-three SU(3) tensors, the totally antisymmetric structure constant f^{abc} and the totally symmetric d^{abc} tensor. Either one or the other will appear as a global factor depending on the transformation properties of the GF under time reversal.

The diagrams giving rise to the gluonic corrections are shown in figure 2. Diagrams (h), (i) and (j) are analogous to those in figure 1 (c), (e) and (f), respectively. Diagrams (d) and (e) are IR and UV safe, even though they have gluons attached to zero momentum quark lines. Much as we did in section 3, we regularise those divergences in dimensional regularisation, renormalise the quark condensate as in the last section and after summing up all diagrams IR divergences will cancel out. The rest of the (UV) divergences are absorbed in counterterms as usual, resulting in a finite (but scale dependent) result.

Loop corrections manifest themselves as logarithms, dilogarithms and constant pieces. In general we will have the following decomposition:

$$C^{\alpha_s}_{\langle \bar{q}q\rangle} = a_s\,\frac{C_F}{8}\left[L_p \log\left(-\frac{p^2}{\mu^2}\right) + L_q \log\left(-\frac{q^2}{\mu^2}\right) + L_r \log\left(-\frac{r^2}{\mu^2}\right) + L_d\, C_0 + L_c\right], \quad (4.1)$$

where $r \equiv -(p+q)$ and the L_i are μ-independent meromorphic functions of the squared external momenta. This simple structure arises because in the chiral limit all internal lines,

[8]In this work we use the convention $\varepsilon^{0123} = +1$.

either quark or gluon, are massless. The massless, scalar three-point integral $C_0(p^2,q^2,r^2)$ collects all dilogarithms and its explicit expression reads [34]:

$$C_o\left(p^2,q^2,r^2\right) \equiv -\,i\,(4\pi)^2\int\frac{\mathrm{d}^4k}{(2\pi)^4}\,\frac{1}{k^2(p+k)^2(q-k)^2} \tag{4.2}$$
$$= \frac{1}{\sqrt{\lambda}}\,\Bigg\{\mathrm{Li}_2\left(-\frac{\lambda+q^2+p^2-r^2}{\lambda-q^2-p^2+r^2}\right)-\,\mathrm{Li}_2\left(-\frac{\lambda-q^2-p^2+r^2}{\lambda+q^2+p^2-r^2}\right)$$
$$+\,\mathrm{Li}_2\left(-\frac{\lambda+q^2+r^2-p^2}{\lambda-q^2-r^2+p^2}\right)-\,\mathrm{Li}_2\left(-\frac{\lambda-q^2-r^2+p^2}{\lambda+q^2+r^2-p^2}\right)$$
$$+\,\mathrm{Li}_2\left(-\frac{\lambda+r^2+p^2-q^2}{\lambda-r^2-p^2+q^2}\right)-\,\mathrm{Li}_2\left(-\frac{\lambda-r^2-p^2+q^2}{\lambda+r^2+p^2-q^2}\right)\Bigg\}\,,$$

where λ is the well-known Källen function $\lambda\left(p^2,q^2,r^2\right)=\left(p^2+q^2-r^2\right)^2-4\,p^2q^2$. Up to order α_s, the μ dependence of $C_{\langle\bar{q}q\rangle}$ then corresponds to

$$-\,\mu\,\frac{\mathrm{d}}{\mathrm{d}\mu}\,C_{\langle\bar{q}q\rangle}(\mu)=a_s\frac{C_F}{4}\,(L_p+L_q+L_r)\,. \tag{4.3}$$

From the general expression eq. (4.1) we see that there is no possible choice of μ that makes all logarithms cancel simultaneously. Then we are forced to include an infinite number of resonances in order to match onto RχT. Moreover, besides logarithms we also have the (μ-independent) dilogarithms, and it is unclear how to match those to the sum of resonance contributions. Let us next explore the different sectors.

4.1 Zero-rank GFs: $\langle$SSS$\rangle$ and $\langle$SPP$\rangle$

Before starting the discussion of this sector we remind the reader that GFs of the type $\langle SSP\rangle$ or $\langle PPP\rangle$ are forbidden by parity invariance of the strong interactions. The Lorentz structure of this sector is trivial, as it has no Lorentz index; time reversal symmetry fully determines the flavour structure:

$$\Pi^{abc}_{SSS(SPP)}(p^2,q^2,r^2)=d^{abc}\,\Pi_{SSS(SPP)}(p^2,q^2,r^2)\,, \tag{4.4}$$

Bose symmetry requires $\Pi_{SSS}(p^2,q^2,r^2)$ to be totally symmetric in its three arguments and also $\Pi_{SPP}(p^2,q^2,r^2)=\Pi_{SPP}(p^2,r^2,q^2)$.

The total anomalous dimension of these GFs is $\gamma=3\,\gamma_S=-\,3\,\gamma_m$, as discussed in section 2. Thus when calculating them in RχT (or χPT) one finds that they are proportional to $\langle\bar{q}q\rangle^3$.[9] On the other hand in the OPE side the first non-vanishing contribution comes from the (single) quark condensate. The RGE for its Wilson coefficient, eq. (2.8) is then

$$\left(\mu\,\frac{\partial}{\partial\mu}\,-2\,\gamma_m\right)C^{SSS(SPP)}_{\langle\bar{q}q\rangle}(\mu)=0\,, \tag{4.5}$$

[9]In χPT with its hadronic degrees of freedom, there is no dynamical generation of the QCD renormalisation scale μ (i.e. it does not come from the UV divergences of Feynman diagrams) and thus it is fully contained in $B_0=-\,\langle\bar{q}q\rangle/F^2$, the quark masses and unphysical couplings like H_1^r and H_2^r. Furthermore, all LEC of the additional operators in the chiral Lagrangian which arise in the presence of tensor sources [29] also depend on μ. The QCD renormalisation scale μ is not to be confused with the chiral renormalisation scale μ_χ, showing up when computing chiral logarithms.

JHEP04(2008)040

where the Wilson coefficients $C^{SSS(SPP)}_{\langle\bar{q}q\rangle}(\mu)$ have been defined as

$$\Pi_{SSS(SPP)}(p^2,q^2,r^2,\mu) = C^{SSS(SPP)}_{\langle\bar{q}q\rangle}(p^2,q^2,r^2,\mu)\langle\bar{q}q\rangle(\mu)\,. \tag{4.6}$$

At $\mathcal{O}(\alpha_s^0)$ their expressions read:

$$C^{SSS}_{\langle\bar{q}q\rangle} = -\frac{2}{p^2\,q^2\,r^2}\,\lambda\left(p^2,q^2,r^2\right)\,, \qquad C^{SPP}_{\langle\bar{q}q\rangle} = \frac{2}{p^2\,q^2\,r^2}\left[p^4 - \left(q^2-r^2\right)^2\right]\,. \tag{4.7}$$

Our explicit results for the $\mathcal{O}(\alpha_s)$ corrections to these expressions (and for the remaining Wilson coefficients) are rather lengthy and thus have been relegated to appendix B.

In ref. [19], the leading order expressions (4.7) have been matched to hadronic representations which took into account constraints arising from χPT and RχT, thereby allowing to estimate some of the LECs in these frameworks. Going to the next-to-leading order, now we have to face two complications. First of all, we have the question of scale dependence of the GFs. This problem could be treated like the $\langle VT\rangle$ GF of the last section: by defining quantities which are explicitly scale independent through multiplication with appropriate scale factors. (This problem cannot even be addressed at the leading order.) Secondly, due to the presence of logarithms (and dilogarithms) the matching definitely requires an infinite tower of resonances, making it much more involved. For this reason, we postpone a detailed discussion of the matching procedure for three-point order-parameter GFs to a forthcoming publication.[10]

4.2 Odd-intrinsic-parity sector: $\langle$VVP$\rangle$, $\langle$AAP$\rangle$ and $\langle$VAS$\rangle$ GFs

These GFs are rank-two Lorentz tensors and so in principle they can be written in terms of many independent Lorentz tensors built from external momenta, the metric tensor $g^{\mu\nu}$ and the Levi-Civita symbol $\varepsilon^{\mu\nu\alpha\beta}$. However chiral Ward identities imply $p_\mu\,\Pi^{\mu\nu}_{123} = q_\nu\,\Pi^{\mu\nu}_{123} = 0$. Besides, the transformation properties of the odd-intrinsic-parity sector can only accommodate a Levi-Civita symbol contracted with the two external momenta. Together with time reversal invariance we are left with the following structure:

$$\left(\Pi_{VVP(AAP)[VAS]}\right)^{abc}_{\mu\nu}(p,q) = \Pi_{VVP(AAP)[VAS]}\left(p^2,q^2,r^2\right)\,\varepsilon_{\mu\nu\alpha\beta}\,p^\alpha q^\beta d^{abc}\left(d^{abc}\right)\left[f^{abc}\right]\,. \tag{4.8}$$

Bose symmetry implies $\Pi_{VVP(AAP)}(p^2,q^2,r^2) = \Pi_{VVP(AAP)}(q^2,p^2,r^2)$. The total anomalous dimension of these currents is $\gamma = \gamma_S = -\gamma_m$, and this means that both in RχP (χPT) and leading order OPE the result is proportional to a single quark condensate. Moreover, the RGE for the Wilson coefficient eq. (2.8) reduces to the fact that it is μ-independent:

$$\frac{\mathrm{d}}{\mathrm{d}\mu}\,C^{VVP(AAP)[VAS]}_{\langle\bar{q}q\rangle} = 0\,. \tag{4.9}$$

Analogously to the $\langle SSS\rangle$ and $\langle SPP\rangle$ GFs, we then define the Wilson coefficients as

$$\Pi_{VVP(AAP)[VAS]}(p^2,q^2,r^2,\mu) = C^{VVP(AAP)[VAS]}_{\langle\bar{q}q\rangle}(p^2,q^2,r^2)\,\langle\bar{q}q\rangle(\mu)\,. \tag{4.10}$$

[10] An additional complication in satisfying quark-counting rules that requires the matching of higher-point functions with an infinite set of resonances is discussed in ref. [35].

JHEP04(2008)040

At lowest order they are given by

$$C_{\langle\bar{q}q\rangle}^{VVP} = \frac{p^2+q^2+r^2}{q^2\,p^2\,r^2}\,, \quad C_{\langle\bar{q}q\rangle}^{AAP} = \frac{p^2+q^2-r^2}{q^2\,p^2\,r^2}\,, \quad C_{\langle\bar{q}q\rangle}^{VAS} = \frac{p^2-q^2-r^2}{q^2\,p^2\,r^2}\,. \tag{4.11}$$

The next-to-leading order α_s corrections are again presented in appendix B. Here a technical comment is in order. Since we work in dimensional regularisation, there is a question how to treat γ_5. In all our computations we have employed a fully anticommuting γ_5. Either two γ_5's appear in a trace and can be cancelled before taking the trace, or, in the odd-parity sector, we can first perform the γ contractions before taking the trace, and are then left with traces of only four γ-matrices and a γ_5 which are unambiguous. As expected, in the odd-parity sector our Wilson coefficients turn out to be scale independent.

4.3 Even-intrinsic-parity sector: $\langle$VVS$\rangle$, $\langle$AAS$\rangle$ and $\langle$VAP$\rangle$ GFs

These last GFs are also rank-two tensors and so they can be built out of many Lorentz structures as well. The chiral Ward identities are not so simple as in the odd-intrinsic-parity sector and read:

$$\begin{aligned}
p^\mu\,(q^\nu)\,(\Pi_{VVS})^{abc}_{\mu\nu}(p,q) &= 0\,,\\
p^\mu\,(q^\nu)\,(\Pi_{AAS})^{abc}_{\mu\nu}(p,q) &= 2\,\langle\bar{q}q\rangle\,d^{abc}\,\frac{q_\nu}{q^2}\left(\frac{p_\mu}{p^2}\right),\\
p^\mu\,(\Pi_{VAP})^{abc}_{\mu\nu}(p,q) &= -\,2\,\langle\bar{q}q\rangle\,f^{abc}\left(\frac{q_\nu}{q^2}+\frac{r_\nu}{r^2}\right),\\
q^\nu\,(\Pi_{VAP})^{abc}_{\mu\nu}(p,q) &= 2\,\langle\bar{q}q\rangle\,f^{abc}\,\frac{r_\mu}{r^2}\,,
\end{aligned} \tag{4.12}$$

where we have also used time reversal invariance. These results rely on the fact that the $\langle AP\rangle$ GF is fully determined by the chiral Ward identity eq. (3.2), and they must be satisfied in any sensible description of the strong interactions. For instance, concentrating on $\langle AAP\rangle$ and $\langle VAS\rangle$, in RχT this means that once we contract with one momenta, only the pion pole can survive. In the OPE they also have deep implications: since there is no other condensate than $\langle\bar{q}q\rangle$ in eq. (4.12), it implies that the contribution of higher dimension condensates vanishes when contracting with one external momenta, and since there is no α_s factor all contributions to the $\langle\bar{q}q\rangle$ beyond leading order must also vanish when the contraction is performed. eq. (4.12) allows us to write the GFs as

$$\begin{aligned}
\left(\Pi^{\mu\nu}_{VVS}\right)^{abc}(q^2,p^2,r^2) &= d^{abc}\Big[P^{\mu\nu}(p,q)\,\mathcal{F}_{VVS}(p^2,q^2,r^2)+Q^{\mu\nu}(p,q)\,\mathcal{G}_{VVS}(p^2,q^2,r^2)\Big],\\
\left(\Pi^{\mu\nu}_{AAS}\right)^{abc}(q^2,p^2,r^2) &= f^{abc}\Big[P^{\mu\nu}(p,q)\,\mathcal{F}_{AAS}(p^2,q^2,r^2)+Q^{\mu\nu}(p,q)\,\mathcal{G}_{AAS}(p^2,q^2,r^2)\\
&\qquad +2\,\langle\bar{q}q\rangle\,\frac{p^\mu\,q^\nu}{q^2\,p^2}\Big],\\
\left(\Pi^{\mu\nu}_{VAP}\right)^{abc}(q^2,p^2,r^2) &= f^{abc}\Big[P^{\mu\nu}(p,q)\,\mathcal{F}_{VAP}(p^2,q^2,r^2)+Q^{\mu\nu}(p,q)\,\mathcal{G}_{VAP}(p^2,q^2,r^2)\\
&\qquad -2\,\langle\bar{q}q\rangle\left(\frac{(p+2\,q)^\mu\,q^\nu}{q^2\,r^2}-\frac{g^{\mu\nu}}{r^2}\right)\Big],
\end{aligned} \tag{4.13}$$

where the transverse tensors $P^{\mu\nu}$ and $Q^{\mu\nu}$ are defined as

$$\begin{aligned} P^{\mu\nu}(p,q) &\equiv q^\mu\, p^\nu - (p\cdot q)\, g^{\mu\nu}\,, \\ Q^{\mu\nu}(p,q) &\equiv p^2\, q^\mu\, q^\nu + q^2\, p^\mu\, p^\nu - (p\cdot q)\, p^\mu\, q^\nu - p^2\, q^2\, g^{\mu\nu}\,. \end{aligned} \tag{4.14}$$

$\mathcal{F}$ and $\mathcal{G}$ are two scalar functions satisfying Bose symmetry in the first two arguments for the $\langle VVS\rangle$ and $\langle AAS\rangle$ GFs:

$$\mathcal{F}\,[\mathcal{G}]_{VVS\,(AAS)}\left(p^2,q^2,r^2\right) = \mathcal{F}\,[\mathcal{G}]_{VVS\,(AAS)}\left(q^2,p^2,r^2\right)\,. \tag{4.15}$$

At lowest order the determination of the scalar functions $\mathcal{F}$ and $\mathcal{G}$ is straightforward, but once we go beyond this level their direct computation turns out to be rather complicated. Instead it will be much simpler to concentrate on the determination of linear combinations of these factors, obtained by taking appropriate contractions in eq. (4.13) (i.e. those contractions that do not reduce to a Ward identity of eq. (4.12)). They take the form

$$\begin{aligned} g_{\mu\nu}\,\Pi^{\mu\nu}_{VVS} &= \frac{3}{2}\left(p^2+q^2-r^2\right)\mathcal{F}_{VVS} - \left(\frac{\lambda}{4}+3\,p^2q^2\right)\mathcal{G}_{VVS}\,, \\ q_\mu\,p_\nu\Pi^{\mu\nu}_{VVS} &= -\frac{\lambda}{4}\,\mathcal{F}_{VVS} + \frac{\lambda}{8}\left(p^2+q^2-r^2\right)\mathcal{G}_{VVS}\,, \\ g_{\mu\nu}\,\Pi^{\mu\nu}_{AAS} &= \frac{3}{2}\left(p^2+q^2-r^2\right)\mathcal{F}_{AAS} - \left(\frac{\lambda}{4}+3\,p^2q^2\right)\mathcal{G}_{AAS} - \langle\bar{q}q\rangle\,\frac{\left(p^2+q^2-r^2\right)}{p^2\,q^2}\,, \\ q_\mu\,p_\nu\,\Pi^{\mu\nu}_{AAS} &= -\frac{\lambda}{4}\,\mathcal{F}_{AAS} + \frac{\lambda}{8}\left(p^2+q^2-r^2\right)\mathcal{G}_{AAS} + \langle\bar{q}q\rangle\,\frac{\left(p^2+q^2-r^2\right)^2}{2\,p^2\,q^2}\,, \\ g_{\mu\nu}\,\Pi^{\mu\nu}_{VAP} &= \frac{3}{2}\left(p^2+q^2-r^2\right)\mathcal{F}_{VAP} - \left(\frac{\lambda}{4}+3\,p^2q^2\right)\mathcal{G}_{VAP} + \langle\bar{q}q\rangle\,\frac{\left(p^2+5\,q^2-r^2\right)}{q^2\,r^2}\,, \\ q_\mu\,p_\nu\,\Pi^{\mu\nu}_{VAP} &= -\frac{\lambda}{4}\,\mathcal{F}_{VAP} + \frac{\lambda}{8}\left(p^2+q^2-r^2\right)\mathcal{G}_{VAP} - \langle\bar{q}q\rangle\,\frac{\left(p^2-r^2\right)^2-q^4}{2\,q^2\,r^2}\,. \end{aligned} \tag{4.16}$$

Once these contractions are known, we are in a position to determine the scalar functions $\mathcal{F}$ and $\mathcal{G}$ by inverting eqs. (4.16). The contractions have the same symmetry properties under exchange of momenta as $\mathcal{F}$ and $\mathcal{G}$, due to Bose symmetry. eqs. (4.16) admit an expansion in α_s, and the terms proportional to $\langle\bar{q}q\rangle$ contribute only at zeroth order in α_s. Thus, for the radiative corrections we are concerned about, the third and fifth lines reduce to the first, while the fourth and sixth reduce to the second. The same discussion concerning RχT and the OPE as in the odd-intrinsic-parity sector applies also here, and again Wilson coefficients turn out to be scale independent. We define those Wilson coefficients as

$$\begin{aligned} g_{\mu\nu}\,\Pi^{\mu\nu}_{VVS(AAS)[VAP]}(\mu) &= C^{g\,VVS(AAS)[VAP]}_{\langle\bar{q}q\rangle}(p^2,q^2,r^2)\langle\bar{q}q\rangle(\mu)\,, \\ q_\mu\,p_\nu\,\Pi^{\mu\nu}_{VVS(AAS)[VAP]}(\mu) &= C^{qp\,VVS(AAS)[VAP]}_{\langle\bar{q}q\rangle}(p^2,q^2,r^2)\langle\bar{q}q\rangle(\mu)\,. \end{aligned} \tag{4.17}$$

The leading-order results for these coefficients are found to be:

$$
\begin{aligned}
C^{g\,VVS}_{\langle\bar{q}q\rangle} &= \frac{1}{p^2\,q^2\,r^2}\left[\left(p^2-q^2\right)^2+\left(p^2+q^2\right)r^2-2\,r^4\right], \quad C^{qp\,VVS}_{\langle\bar{q}q\rangle} = -\,\frac{1}{2\,p^2\,q^2}\,\lambda\,,\\
C^{g\,AAS}_{\langle\bar{q}q\rangle} &= \frac{1}{p^2\,q^2\,r^2}\left[\left(p^2-q^2\right)^2-3\left(p^2+q^2\right)r^2+2\,r^4\right], \quad C^{qp\,AAS}_{\langle\bar{q}q\rangle} = \frac{1}{2\,p^2\,q^2}\left(\lambda+4\,p^2q^2\right),\\
C^{g\,VAP}_{\langle\bar{q}q\rangle} &= -\,\frac{1}{p^2\,q^2\,r^2}\left(p^2-q^2-2\,r^2\right)\left(p^2+q^2-r^2\right),\\
C^{qp\,VAP}_{\langle\bar{q}q\rangle} &= -\,\frac{1}{2\,p^2\,q^2\,r^2}\left[r^2\lambda+2\,p^2\,q^2\left(p^2-q^2+r^2\right)\right]. \qquad (4.18)
\end{aligned}
$$

Like before, the complete results of our computation at order α_s can be found in appendix B.

5. Conclusions

In this work we have made a step forward in our understanding of quark-hadron duality, by computing order α_s corrections to the leading quark condensate contribution for 2- and 3-point Green functions which are order parameters of the chiral symmetry breaking. We have concentrated on these order-parameter GFs, since in this case the matching between the OPE result and the corresponding GFs obtained in a low-energy effective theory like χPT or RχT is more transparent.

The matching further simplifies in the large-N_C limit of QCD, because then, the hadronic spectrum only consists of an infinite set of zero-width resonances. Still, generally, at the next-to-leading order, the appearance of logarithms in the kinematical variables requires the inclusion of the full infinite number of those resonances in order to be able to reproduce the logarithms present on the OPE side.

For the 2-point $\langle VT\rangle$ GF, we have discussed the matching in more detail. The inclusion of α_s corrections allows to trace the dependence of both, the OPE and the hadronic side, on the short-distance renormalisation scale μ. These μ dependencies show up in the quark condensate as well as possible anomalous dimensions of the initial currents on the OPE side, while in the low-energy theory, besides the condensate, the LECs of the tensor sources explicitly depend on μ [29]. To be able to do some numerics, finally we have assumed the minimal-hadronic-ansatz as a crude approximation, and have matched the $\langle VT\rangle$ GF with a single multiplet of vector-meson resonances. This allowed to estimate the vector-meson tensor coupling f_V^T.

The main and most involved aim of our work was the calculation of the α_s corrections for the leading OPE term of 3-point order-parameter GFs. Details of the computation for the different types of 3-point GFs to be considered have been discussed in section 4, while the rather lengthy final results have been relegated to appendix B.

Additional complications for the matching in the case of 3-point GFs arise from two facts: firstly, due to the presence of three independent kinematical variables, no choice of the renormalisation scale μ allows to resum all of the logarithms which are present in our results. Secondly, in addition in the next-to-leading order result also dilogarithms arise and it is unclear how to match them even with an infinite set of resonances on the hadronic

JHEP04(2008)040

side. Therefore, we postpone a detailed discussion of the matching in the case of 3-point GFs to forthcoming work.

Acknowledgments

We would would like to thank Santi Peris and Toni Pich for useful discussions. This work has been supported in part by the EU Contract No. MRTN-CT-2006-035482 (FLAVIAnet) (MJ, VM), by CICYT-FEDER-FPA2005-02211 (MJ), by MEC (Spain) grant No. FPA2007-60323 (VM) and by Generalitat Valenciana grant No. GVACOMP2007-156 (VM).

A. Non-renormalisation of the $\langle$AP$\rangle$ GF

In this appendix we demonstrate that in the chiral limit Ward identities completely fix the $\langle AP\rangle$ GF, to be fully saturated by single pion exchange. Lorentz and SU(3) invariance imply

$$(\Pi_{\rm AP})^{ab}_{\mu}(p) = i\,\delta^{ab}\,\Pi_{\rm AP}(p^2)\,p_\mu\,. \tag{A.1}$$

From the definition of time-ordered product and the conservation of the axial-vector current it follows that

$$\partial^\mu\,T\{A^a_\mu(x)\,P^b(0)\} = \delta(x^0)\,[A^a_0(x),\,P^b(0)]\,. \tag{A.2}$$

On the other hand Fourier transformation relates derivatives with external momenta

$$\int \mathrm{d}^4x\,e^{x\cdot p}\,(\partial^\mu + i\,p^\mu)\,\Big\langle\,0\,\Big|\,\mathrm{T}\{A^a_\mu(x)\,P^b(0)\}\Big|\,0\,\Big\rangle = 0\,. \tag{A.3}$$

The right hand side of eq. (A.2) can be easily computed since the Dirac delta function forces the time dependence on both fields to be the same:

$$\delta(x^0-y^0)\,\Big[A^a_0(x),P^b(y)\Big] = -\,i\left\{\frac{2}{n_f}\,\delta^{ab}\,S(x)\;+\;d^{abc}\,S^c(x)\right\}\,\delta^{(4)}(x-y)\,, \tag{A.4}$$

where $S(x)=\sum_{n_f}\bar q_f q_f$ is the flavour singlet scalar current. Then from eqs. (A.1) to (A.5) it follows that

$$p^\mu\,(\Pi_{\rm AP})^{ab}_\mu(p) = i\,\delta^{ab}\,p^2\,\Pi_{\rm AP} = \frac{2\,i}{n_f}\,\delta^{ab}\,\langle\,0\,|\,S(0)\,|\,0\,\rangle \;=\; 2\,i\,\delta^{ab}\,\langle\bar q q\rangle\,, \tag{A.5}$$

where we have used the fact that vacuum is flavour blind. We can identify $\Pi_{\rm AP} = 2\,\langle\bar q q\rangle\,/p^2$ and then demonstrate eq. (3.2).

B. Explicit expression for the Wilson coefficients

In this appendix we present the complete expression for the α_s corrections to the quark condensate Wilson coefficients. We have split our results as shown in eq. (4.1), in terms of coefficients multiplying logarithms, dilogarithms and polynomial terms. The results can be found in table 1.

$\langle SSS \rangle$	
L_p	$\frac{4}{p^2q^2r^2}\left[4\,p^4+q^4+r^4-6\,q^2\,r^2-3\,p^2\left(q^2+r^2\right)\right]$
L_q	$\frac{4}{p^2q^2r^2}\left[4\,q^4+p^4+r^4-6\,p^2\,r^2-3\,q^2\left(p^2+r^2\right)\right]$
L_r	$\frac{4}{p^2q^2r^2}\left[4\,r^4+q^4+p^4-6\,q^2\,p^2-3\,r^2\left(q^2+p^2\right)\right]$
L_d	$\frac{8}{p^2q^2r^2}\left[p^6+q^6+r^6-2\,p^4\left(q^2+r^2\right)-2\,q^4\left(p^2+r^2\right)-2\,r^4\left(p^2+q^2\right)\right]$
L_c	$\frac{8}{p^2q^2r^2}\left(-5\,p^4-5\,q^4-5\,r^4+14\,p^2\,r^2+14\,q^2\,p^2+14\,q^2\,r^2\right)$
$\langle SPP \rangle$	
L_p	$\frac{4}{p^2q^2r^2}\left[r^4+q^4-4\,p^4-3\,p^2\,(r^2+q^2)-6\,r^2\,q^2\right]$
L_q	$\frac{4}{p^2q^2r^2}\left[4\,q^4+r^4-p^4+3\,q^2\left(p^2-r^2\right)\right]$
L_r	$\frac{4}{p^2q^2r^2}\left[4\,r^4+q^4-p^4+3\,r^2\left(p^2-q^2\right)\right]$
L_d	$\frac{8}{p^2q^2r^2}\left[r^6+q^6-p^6-2\,p^2\left(r^4+q^4\right)+2\left(p^4-q^2\,r^2\right)\left(r^2+q^2\right)\right]$
L_c	$\frac{8}{p^2q^2r^2}\left(-5\,r^4-5\,q^4+5\,p^4+14\,r^2\,q^2\right)$
$\langle VVP \rangle$	
L_p	$\frac{-2}{\lambda\,p^2q^2r^2}\left[p^6+r^2\,p^4+p^2\,q^2\left(5\,r^2+q^2\right)-2\left(r^2+q^2\right)\left(r^2-q^2\right)^2\right]$
L_q	$\frac{-2}{\lambda\,p^2q^2r^2}\left[q^6+r^2\,q^4+p^2\,q^2\left(5\,r^2+p^2\right)-2\left(r^2+p^2\right)\left(r^2-p^2\right)^2\right]$
L_r	$\frac{-2}{\lambda\,p^2q^2r^2}\left[\left(p^2+q^2\right)\left(p^2-q^2\right)^2+4\,r^6-\,r^2\left(3\,p^2+q^2\right)\left(3\,q^2+p^2\right)-2\,r^4\left(q^2+p^2\right)\right]$
L_d	$\frac{-4}{\lambda\,p^2q^2r^2}\left[2\left(p^4-3\,p^2q^2+q^4\right)r^4-2\,r^2\left(p^2+q^2\right)\left(p^4+q^4\right)\right.$
	$\left.-2\,r^6\left(p^2+q^2\right)+r^8+\left(p^2-q^2\right)^2\left(p^4+q^4-q^2\,p^2\right)\right]$
L_c	$\frac{2}{p^2q^2r^2}\left(p^2+q^2+4\,r^2\right)$
$\langle AAP \rangle$	
L_p	$\frac{-2}{\lambda\,p^2q^2r^2}\left[p^6-3\,r^2\,p^4+p^2\,q^2\left(5\,r^2+q^2\right)+2\left(r^2-q^2\right)^3\right]$
L_q	$\frac{-2}{\lambda\,p^2q^2r^2}\left[q^6-3\,r^2\,q^4+p^2\,q^2\left(5\,r^2+p^2\right)+2\left(r^2-p^2\right)^3\right]$
L_r	$\frac{-2}{\lambda\,p^2q^2r^2}\left[-4\,r^6+6\,r^4\left(p^2+q^2\right)-r^2\left(q^2+3\,p^2\right)\left(p^2+3\,q^2\right)+\left(p^2-q^2\right)^2\left(p^2+q^2\right)\right]$
L_d	$\frac{-4}{\lambda\,p^2q^2r^2}\left[2\,r^6\left(p^2+q^2\right)+6\,p^2\,q^2\,r^4-2\,r^2\left(p^2+q^2\right)\left(p^4+q^4\right)\right.$

	$- r^8 + \left(p^2 - q^2\right)^2 \left(p^4 + q^4 - q^2\, p^2\right)\Big]$
L_c	$\frac{2}{p^2q^2r^2}\left(p^2 + q^2 - 4\, r^2\right)$
$\langle V\, A\, S\rangle$	
L_p	$\frac{-2}{\lambda\, p^2q^2r^2}\Big[p^6 + 3\, p^4\left(2\, q^2 - r^2\right) - p^2q^2\left(5\, r^2 + q^2\right) + 2\left(r^2 + q^2\right)\left(r^2 - q^2\right)^2\Big]$
L_q	$\frac{-2}{\lambda\, p^2q^2r^2}\left[-2\, p^6 + \left(q^2 + 6\, r^2\right) p^4 - \left(6\, q^4 + 6\, r^4 - 5\, q^2\, r^2\right) p^2 - q^6 + 2\, r^6 - q^4\, r^2\right]$
L_r	$\frac{-2}{\lambda\, p^2q^2r^2}\left[p^6 + 7q^2\, p^2(q^2 - p^2) - q^6 - 4\, r^6 + 2\left(3\, p^2 + q^2\right) r^4 - 3\left(p^4 - q^4\right) r^2\right]$
L_d	$\frac{-4}{\lambda\, p^2q^2r^2}\Big[\left(3\, q^6 + 2\, r^2\, q^4 + 6\, r^4\, q^2 + 2\, r^6\right) p^2 - \left(r^2 - q^2\right)^2\left(r^4 + q^4\right)$
	$+\ p^8 - p^6\left(3\, q^2 + 2\, r^2\right) - 2\, r^2\, q^2\, p^4\Big]$
L_c	$\frac{2}{p^2q^2r^2}\left(p^2 - q^2 - 4\, r^2\right)$
$g_{\mu\nu}\,\langle V^\mu V^\nu S\rangle$	
L_p	$\frac{2}{p^2q^2r^2}\left[2\left(q^4 - r^4\right) - p^4 + p^2\left(r^2 - 3\, q^2\right)\right]$
L_q	$\frac{2}{p^2q^2r^2}\left[2\left(p^4 - r^4\right) - q^4 + q^2\left(r^2 - 3\, p^2\right)\right]$
L_r	$\frac{2}{p^2q^2r^2}\left[6\, p^2\, q^2 + 4\, r^4 - p^4 - q^4 - r^2\left(p^2 + q^2\right)\right]$
L_d	$\frac{4}{p^2q^2r^2}\left[2\, r^6 - \left(r^4 + p^4 + q^4 - 3\, p^2\, q^2\right)\left(p^2 + q^2\right)\right]$
L_c	$\frac{4}{p^2q^2r^2}\left[3\, r^2(p^2 + q^2 - r^2) - 4\, p^2\, q^2\right]$
$q_\mu\, p_\nu\,\langle V^\mu V^\nu S\rangle$	
L_p	$\frac{1}{q^2r^2}\left[q^2\left(2\, q^2 - 2\, p^2 + r^2\right) - 3\, r^2\left(p^2 - r^2\right)\right]$
L_q	$\frac{1}{p^2r^2}\left[p^2\left(2\, p^2 - 2\, q^2 + r^2\right) - 3\, r^2\left(q^2 - r^2\right)\right]$
L_r	$\frac{1}{p^2q^2}\left[3\left(p^4 + q^4\right) - 3\, r^2\left(p^2 + q^2\right) - 2\, p^2\, q^2\right]$
L_d	$\frac{2}{p^2q^2}\left[p^6 - p^4\, r^2 + (q^2 - r^2)^2(q^2 + r^2) - p^2\, r^2(4\, q^2 + r^2)\right]$
L_c	$\frac{1}{q^2r^2p^2}\left(p^2 + q^2 - 2\, r^2\right)\lambda$
$g_{\mu\nu}\,\langle A^\mu A^\nu S\rangle$	
L_p	$\frac{2}{p^2q^2r^2}\Big[2\left(q^2 - r^2\right)^2 - p^4 + p^2\left(5\, r^2 - 3\, q^2\right)\Big]$
L_q	$\frac{2}{p^2q^2r^2}\Big[2\left(p^2 - r^2\right)^2 - q^4 + q^2\left(5\, r^2 - 3\, p^2\right)\Big]$
L_r	$\frac{2}{p^2q^2r^2}\left[6\, p^2\, q^2 - 4\, r^4 - p^4 - q^4 - r^2\left(p^2 + q^2\right)\right]$

L_d	$-\frac{4}{p^2q^2r^2}\left[2\,r^6+\left(-3\,r^4+p^4+q^4-3\,p^2\,q^2\right)\left(p^2+q^2\right)\right]$
L_c	$\frac{4}{p^2q^2r^2}\left[3\,r^4-4\,p^2\,q^2-3\,r^2\left(q^2+p^2\right)\right]$
$q_\mu\,p_\nu\,\langle A^\mu A^\nu S\rangle$	
L_p	$\frac{1}{q^2r^2}\left[r^2\left(q^2-3\,p^2+3\,r^2\right)+2\,q^2\left(q^2-p^2\right)\right]$
L_q	$\frac{1}{p^2r^2}\left[r^2\left(p^2-3\,q^2+3\,r^2\right)-2\,p^2\left(q^2-p^2\right)\right]$
L_r	$\frac{1}{p^2q^2}\left[-2\,p^2\,q^2+3\,(p^4+q^4)-3\,r^2(p^2+q^2)\right]$
L_d	$\frac{2}{p^2q^2}\left[p^6-3\,p^4\,r^2+3\,p^2\,r^4+(q^2-r^2)^3\right]$
L_c	$\frac{1}{p^2q^2r^2}\left(p^2+q^2+2\,r^2\right)\lambda$
$g_{\mu\nu}\,\langle V^\mu A^\nu P\rangle$	
L_p	$\frac{2}{p^2q^2r^2}\left[2\left(q^4-r^4\right)+p^4-p^2\left(5\,r^2+q^2\right)\right]$
L_q	$\frac{2}{p^2q^2r^2}\left[p^2\left(q^2+4\,r^2\right)-2\,p^4-q^4-2\,r^4+q^2\,r^2\right]$
L_r	$\frac{2}{p^2q^2r^2}\left[p^4-q^4+4\,r^4+r^2\left(p^2-q^2\right)\right]$
L_d	$\frac{4}{p^2q^2r^2}\left[p^6-2\,q^2\,p^4+2\,p^2\,q^4-q^6+2\,r^6-r^4\left(3\,p^2+q^2\right)\right]$
L_c	$\frac{12}{p^2q^2}\left(q^2+p^2-r^2\right)$
$q_\mu\,p_\nu\,\langle V^\mu A^\nu P\rangle$	
L_p	$\frac{3}{q^2}\left(p^2+3\,q^2-r^2\right)$
L_q	$-\frac{3}{p^2}\left(3\,p^2+q^2-r^2\right)$
L_r	$\frac{3}{p^2q^2}\left(q^2-p^2\right)\left(q^2+p^2-r^2\right)$
L_d	$-\frac{2}{p^2q^2}\left[p^6-3\,r^2\,p^4+p^2\,r^2\left(2\,q^2+3\,r^2\right)-\left(q^2-r^2\right)^2\left(q^2+r^2\right)\right]$
L_c	$-\frac{1}{p^2q^2r^2}\left(p^2-q^2+2\,r^2\right)\lambda$

Table 1: α_s corrections to the Wilson coefficients for three-point GFs.

References

[1] S. Weinberg, *Phenomenological Lagrangians*, *Physica* **A 96** (1979) 327.

[2] J. Gasser and H. Leutwyler, *Chiral perturbation theory to one loop*, *Ann. Phys. (NY)* **158** (1984) 142.

[3] J. Gasser and H. Leutwyler, *Chiral perturbation theory: expansions in the mass of the strange quark*, *Nucl. Phys.* **B 250** (1985) 465.

[4] K.G. Wilson, *Nonlagrangian models of current algebra*, *Phys. Rev.* **179** (1969) 1499.

[5] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *QCD and resonance physics. Sum rules*, *Nucl. Phys.* **B 147** (1979) 385.

[6] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *QCD and resonance physics: applications*, *Nucl. Phys.* **B 147** (1979) 448.

[7] G. 't Hooft, *A planar diagram theory for strong interactions*, *Nucl. Phys.* **B 72** (1974) 461.

[8] G. 't Hooft, *A two-dimensional model for mesons*, *Nucl. Phys.* **B 75** (1974) 461.

[9] E. Witten, *Baryons in the* $1/N$ *expansion*, *Nucl. Phys.* **B 160** (1979) 57.

[10] G. Ecker, J. Gasser, A. Pich and E. de Rafael, *The role of resonances in chiral perturbation theory*, *Nucl. Phys.* **B 321** (1989) 311.

[11] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, *Chiral Lagrangians for massive spin* 1 *fields*, *Phys. Lett.* **B 223** (1989) 425.

[12] B. Moussallam, *A sum rule approach to the violation of Dashen's theorem*, *Nucl. Phys.* **B 504** (1997) 381 [hep-ph/9701400].

[13] S. Peris, M. Perrottet and E. de Rafael, *Matching long and short distances in large-*N_c *QCD*, *JHEP* **05** (1998) 011 [hep-ph/9805442].

[14] P. Masjuan and S. Peris, *A rational approach to resonance saturation in large-*N_c *QCD*, *JHEP* **05** (2007) 040 [arXiv:0704.1247].

[15] J. Mondejar and A. Pineda, *Constraints on Regge models from perturbation theory*, *JHEP* **10** (2007) 061 [arXiv:0704.1417].

[16] M. Knecht and A. Nyffeler, *Resonance estimates of* $O(p^6)$ *low-energy constants and QCD short-distance constraints*, *Eur. Phys. J.* **C 21** (2001) 659 [hep-ph/0106034].

[17] P.D. Ruiz-Femenia, A. Pich and J. Portolés, *Odd-intrinsic-parity processes within the resonance effective theory of QCD*, *JHEP* **07** (2003) 003 [hep-ph/0306157].

[18] V. Cirigliano, G. Ecker, M. Eidemüller, A. Pich and J. Portolés, *The* $\langle VAP \rangle$ *Green function in the resonance region*, *Phys. Lett.* **B 596** (2004) 96 [hep-ph/0404004].

[19] V. Cirigliano et al., *The* $\langle SPP \rangle$ *Green function and* SU(3) *breaking in* K_{l3} *decays*, *JHEP* **04** (2005) 006 [hep-ph/0503108].

[20] V. Cirigliano et al., *Towards a consistent estimate of the chiral low-energy constants*, *Nucl. Phys.* **B 753** (2006) 139 [hep-ph/0603205].

[21] V. Mateu and J. Portolés, *Form factors in radiative pion decay*, *Eur. Phys. J.* **C 52** (2007) 325 [arXiv:0706.1039].

[22] V.P. Spiridonov and K.G. Chetyrkin, *Nonleading mass corrections and renormalization of the operators* $m\bar{\psi}\psi$ *and* $G^2_{\mu\nu}$, *Sov. J. Nucl. Phys.* **47** (1988) 522 [*Yad. Fiz.* **47** (1988) 818].

[23] M. Jamin and M. Münz, *Current correlators to all orders in the quark masses*, *Z. Physik* **C 60** (1993) 569 [hep-ph/9208201].

[24] M. Jamin, *Flavour-symmetry breaking of the quark condensate and chiral corrections to the Gell-Mann-Oakes-Renner relation*, *Phys. Lett.* **B 538** (2002) 71 [hep-ph/0201174].

[25] A.J. Buras, M. Jamin and P.H. Weisz, *Leading and next-to-leading QCD corrections to ε-parameter and B_0-$\bar{B}_0$ mixing in the presence of a heavy top quark*, *Nucl. Phys.* **B 347** (1990) 491.

[26] D.J. Broadhurst and A.G. Grozin, *Matching QCD and HQET heavy - light currents at two loops and beyond*, *Phys. Rev.* **D 52** (1995) 4082 [hep-ph/9410240].

[27] J.A. Gracey, *Three loop $\overline{\mathrm{MS}}$ tensor current anomalous dimension in QCD*, *Phys. Lett.* **B 488** (2000) 175 [hep-ph/0007171].

[28] O. Catà and V. Mateu, *Novel patterns for vector mesons from the large-N_C limit*, arXiv:0801.4374.

[29] O. Catà and V. Mateu, *Chiral perturbation theory with tensor sources*, *JHEP* **09** (2007) 078 [arXiv:0705.2948].

[30] P. Ball and V.M. Braun, *The ρ meson light-cone distribution amplitudes of leading twist revisited*, *Phys. Rev.* **D 54** (1996) 2182 [hep-ph/9602323].

[31] V.M. Braun et al., *A lattice calculation of vector meson couplings to the vector and tensor currents using chirally improved fermions*, *Phys. Rev.* **D 68** (2003) 054501 [hep-lat/0306006].

[32] S. Narison and N. Paver, *On some three meson vertex sum rules in quantum chromodynamics*, *Z. Physik* **C 22** (1984) 69.

[33] S. Narison, *On the two photon width of the $\delta(980)$*, *Phys. Lett.* **B 175** (1986) 88.

[34] G. 't Hooft and M.J.G. Veltman, *Scalar one loop integrals*, *Nucl. Phys.* **B 153** (1979) 365.

[35] J. Bijnens, E. Gámiz, E. Lipartia and J. Prades, *QCD short-distance constraints and hadronic approximations*, *JHEP* **04** (2003) 055 [hep-ph/0304222].

JHEP04(2008)040